TENTATIVE SYLLABUS
of the Low Dimensional Topology Summer School

Statement of Purpose

Low dimensional topology is an emerging field, whose significance is underlined by its intimate connection to physics, by the related Abel Prizes Atiah (2004), Gromov (2009), Milnor (2011), and Fields medals (second only to the Abel Prize in mathematics), like the ones for Donaldson, Friedman, Johns and Witten. Actually, many basic notions, like spin structure or Seiberg-Witten invariant, are coming from physics. The Summer School concentrates on recent developments in low dimensional topology, exploring connections to physics.

The lecture series are aimed to introduce the participants into the latest developments in low dimensional topology. We expect that these lectures will provide tools and ideas for further research of the participants, and ultimately help them to successfully finish their graduate studies and produce a PhD dissertation competitive world wide, and write papers of high impact.

Concerning lecturers, the main idea was to choose internationally respected experts who are relatively young, and hence it is easier to connect with them for the audience during the lectures and the discussions.

Pre-requisites

We assume basic knowledge of manifold topology and algebraic topology. Some basic familiarity with the notions of symplectic structures on smooth manifolds will be also assumed. The school aims to attract graduate students working in one of the areas of the lecture series and recent post-docs widening their horizon in low dimensional topology and to learn more about the interaction of this subject with neighboring fields.

Brief overview of the course content

Key topics are Seiberg-Witten monopoles, Legendrian and transverse knots, Alternating links and J-holomorphic curves. The courses explore recent developments, connections among the topics, relations to physics, and further research directions.

Vera Vertesi: Legendrian and transverse knots

After a short description of Heegaard Floer homologies and contact structures I will define invariants for contact structures and knots in contact structures. Then I will discuss applications of these invariants in proving non-fillability of contact structures and distinguishing transverse knots. Lastly I will discuss possible directions of how to define useful contact invariants in the newly developed bordered Floer homologies.

Klaus Niederkruger: J-holomorphic curves and overtwistedness

Higher dimensional contact topology has recently reached an important milestone with the discovery of overtwistedness by Borman, Eliashberg and Murphy. The course will start by briefly discussing equivalent notions of overtwistedness as explained by Casals, Murphy and Presas. Then, we will explain how J-holomorphic curve techniques (initiated by Gromov, and further developed by Eliashberg) allow us to study the fillability of contact manifolds, and the vanishing of contact homology.

Brendan Owens: Alternating links and 4-dimensional topology

In the course we will discuss rational balls, slice knots, unknotting and unlinking, and the application of Donaldson's theorem and Heegaard Floer theory to these problems, making use of the definite manifolds constructed by Gordon and Litherland. This will be good motivation to make some new progress on some of these questions.

Andras Stipsicz and Francesco Lin: Seiberg-Witten monopoles and spin structures

In the first lectures we will review the construction of monopole Floer homology due to Kronheimer and Mrowka. Then we will focus on the Pin(2)-equivariant version of the theory. This is a Morse theoretic approach to Manolescu's recent Pin(2)-equivariant Seiberg-Witten Floer homology, and it can indeed be used to give an alternative disproof (formally identical to Manolescu's) to the longstanding Triangulation conjecture. Time permitting we will discuss other aspects of this theory.

Bibliography

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